

Controlling Robot Manipulators

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Outline of the Course

- Introduction
 - Robot-Components
 - Manipulators
 - Configuration Space
- Software
 - Player
 - Gazebo
- Position Analysis
 - Vectors in Space
 - Representation of Rigid Bodies
 - Representation of Frame Transformations
 - Roboop
- Forward and Inverse Kinematics
- Modeling of Manipulators
 - DH Parameters
 - GLrooop
- Further topics
 - Trajectory Planning
 - Modern Control Methods

Outline

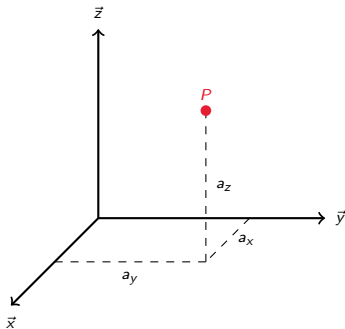
- 1 Vectors in Space
- 2 Representation of Rigid Bodies

Outline

- 1 Vectors in Space
- 2 Representation of Rigid Bodies

Representation of a point in space

- Described by three coordinates relative to a reference frame.
- $P = a_x \vec{x} + a_y \vec{y} + a_z \vec{z}$

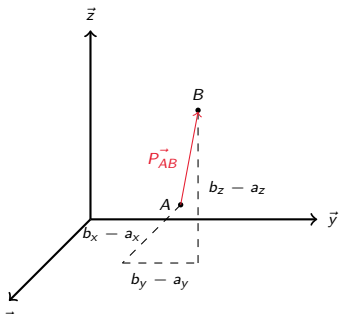


Representation of a vector in space

- Described by three coordinates for the tail and three coordinates for the head relative to a fixed reference frame.

- $$\vec{P}_{AB} = (b_x - a_x)\vec{x} + (b_y - a_y)\vec{y} + (b_z - a_z)\vec{z}$$

- Corresponding matrix form:
$$P_{AB}^{\vec{}} = \begin{pmatrix} b_x - a_x \\ b_y - a_y \\ b_z - a_z \end{pmatrix}$$



Scale factor

- For a more compact notation we will set
 $p_x = b_x - a_x$, $p_y = b_y - a_y$ and $p_z = b_z - a_z$
- A vector can be scaled by a scale factor to **increase or decrease its length**.
- For this we add a fourth component w to the vector, so that it can be written as

$$P'_{AB} = \begin{pmatrix} p'_x \\ p'_y \\ p'_z \\ w \end{pmatrix}, \text{ where } p'_x = p_x * w, p'_y = p_y * w \text{ and } p'_z = p_z * w.$$

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- **Example:** A vector is described as $\vec{P} = 3\vec{x} + 5\vec{y} + 2\vec{z}$. Compute the corresponding vector \vec{P}' with a scale factor of 2.

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- **Example:** A vector is described as $\vec{P} = 3\vec{x} + 5\vec{y} + 2\vec{z}$. Compute the corresponding vector \vec{P}' with a scale factor of 2.
- **Solution:** $\vec{P}' = \begin{pmatrix} 6 \\ 10 \\ 4 \\ 2 \end{pmatrix}$

Unit vector

- The unit vector is a normalized version of a vector that **keeps the orientation**, but **changes the length** so that it equals unity.
- To do this, each component of the vector has to be divided by the square root of the sum of the squares of the three components.

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- **Example:** A vector is described as $\vec{P} = 3\vec{x} + 5\vec{y} + 2\vec{z}$. Compute the corresponding unit vector $P_{unit}^{\vec{}}$.
- **Solution:** $l_P = \sqrt{p_x^2 + p_y^2 + p_z^2} = 6.16$

$$\Rightarrow P_{unit}^{\vec{}} = \begin{pmatrix} 0.487 \\ 0.811 \\ 0.324 \end{pmatrix}$$

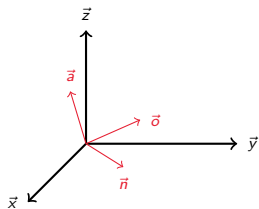
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- 1 Vectors in Space
- 2 Representation of Rigid Bodies**

Representation of a frame at the origin of a fixed reference frame

- A dependend frame that is centered at the origin of a fixed reference frame is described by three unit vectors, which represent the **orientation**.
- The unit vectors are **perpendicular** to each other, and are called \vec{n} , \vec{o} , \vec{a} for normal, orientation and approach.
- Each unit vector in the dependend coordinate frame is represented by its three components in the reference frame.

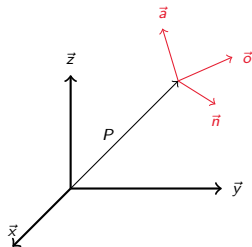
$$F = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}$$



General Representation of a frame in a fixed reference frame

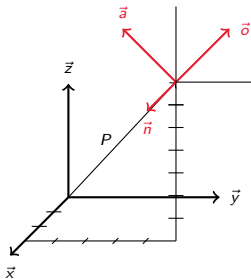
- Beside the rotation, we have to express the **position of the origin** of the dependend frame relative to a fixed reference frame.
- For achieving this, we use a vector that goes from the origin of the reference frame to the origin of the dependend frame. This vector is expressed in the coordinates of the reference frame.

$$F = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



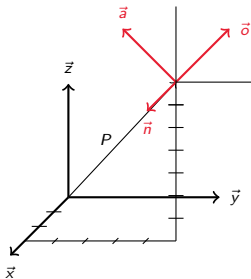
Example

- The dependend frame F shown in the figure is located at $\langle 3, 5, 7 \rangle$. Its n -axis is parallel to the x -axis of the reference frame, its o -axis is the y -axis of the reference frame, rotated by 45° around the x -axis and its a -axis is the z -axis of the reference frame rotated by 45° around the x -axis.
- How can the frame be described in a matrix?



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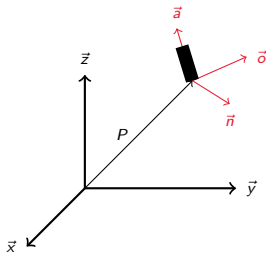


$$F = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0.707 & -0.707 & 5 \\ 0 & 0.707 & 0.707 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of a rigid body

- A rigid body can be represented in space by **attaching a frame**.
- Because the body frame is always fixed in the same position and orientation of the rigid body, the position and orientation of the rigid body relative to the reference frame is always known by comparing its body frame with the reference frame.
- So representing the position and orientation of a rigid body in space is like representing a frame in a fixed reference frame.

$$F_{body} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Representation of a rigid body

- A rigid body is described by **6 values** (DOFs).
- The matrix has 16 values, 3 for positioning, 9 for rotation and 4 for filling up to a 4×4 matrix.
- There are 6 constraints on the 3×3 rotation matrix, so that many matrix configurations are forbidden.
- Each of the 3 **vector's length** must be equal to unity
 $\rightarrow |\vec{n}| = 1, |\vec{o}| = 1, \text{ and } |\vec{a}| = 1.$
- The 3 unit vectors $\vec{n}, \vec{o}, \vec{a}$ are **mutually perpendicular**
 $\rightarrow \vec{n} \cdot \vec{o} = 0, \vec{n} \cdot \vec{a} = 0, \text{ and } \vec{a} \cdot \vec{o} = 0.$
- Instead of the dot product, the **cross product** can be used to express the last constraint
 $\rightarrow \vec{n} \times \vec{o} = \vec{a}, \vec{o} \times \vec{a} = \vec{n}, \text{ and } \vec{a} \times \vec{n} = \vec{o}.$

Representation of a rigid body (cont.)

- As a result of the constraints, the rotation matrix can be defined by 3 values instead of 9 values.
- So for any rotation matrix where 3 values are given, the **remaining 6 values can be computed** by solving the constraint equations.
- It is possible, that there are **more possibilities** for choosing the remaining values.

$$F = \begin{bmatrix} ? & 0 & ? & 5 \\ 0.707 & ? & ? & 3 \\ ? & ? & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow F = \begin{bmatrix} 0.707 & 0 & 0.707 & 5 \\ 0.707 & 0 & -0.707 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ or}$$

$$F = \begin{bmatrix} -0.707 & 0 & -0.707 & 5 \\ 0.707 & 0 & -0.707 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$